

STABLE DISTRIBUTIONS AND PARETIAN TAILS: AUSTRALIAN BOX OFFICE REVENUE EVIDENCE*

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Abstract

Recent empirical literature has identified the importance of information transmission in creating box office blockbusters. It has been shown that box office revenue distributions are characterised by extremities of skew and variance that cannot be explained by such things as expensive budgets, star power, critical acclaim or awards. Empirical research of this industry has subsequently turned its attention to models of information transmission and the consequential implications for box office revenues. Using a new large Australian data set, this paper adds to the literature by exploring the suitability of the α -stable distribution for various definitions of Australian box office revenues.

Keywords: Box Office Revenue, Stable Distributions, Paretian Tails

JEL Classification Numbers: Z11, C16

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I Introduction

Understanding the demand for motion pictures takes economists away from conventional models of demand – the price of consumption for films is typically invariant at specific cinema locations, each film is a perfectly differentiated product and the utility gained from consumption is largely unknown *ex-ante*. When making consumption decisions consumers must rely on signals of product quality which are derived from a variety of sources including advertising, reviews, and shared word of mouth. Earlier empirical models of movie demand have tended to focus on the role of film specific covariates such as production budgets, advertising, awards, reviews, cast appeal, etc. in explaining box office successes.¹ More recently, however, empirical research has recognised a far more important and salient feature of the industry relating to the sharing of information amongst cinemagoers that has the proven ability to create box office ‘blockbusters’ and box office ‘flops’ beyond that attributable to any film specific variable(s). The fact that top grossing movies strongly dominate the mean of box office revenue for both the ‘weekly revenue’ and ‘cumulative revenue’ definitions suggests a heavily right skewed distribution and this feature has led researchers to investigate the underlying dynamic processes that generate the statistical features observed. De Vany and Walls (1996) were the first to empirically explore the extreme events (blockbusters) of the industry in this context when they likened the evolution of the box office revenue distribution to the Bose Einstein allocation model of physics.² Since this seminal contribution, De Vany and Walls (1999, 2004a, 2004b) have extended their research to categorise the box office

¹ Smith and Smith (1986), Prag and Casevant (1994), Eliashberg and Shugan (1997), Albert (1998) and Ravid (1999) provide examples.

² This model bears similarity to the well known Polya-Urn models and informational cascade models. See, for example, Banerjee (1992), Bikhchandani, Hirshleifer and Welch (1992), Chamley and Gale (1994), Smith and Sorenson (2000) and SgROI (2002).

revenue distribution in the ‘stable’ class noted for the features of nesting extremities of skew and variance. Stable distributions were initially discovered by Paul Levy in the 1920s but the lack of closed formulas for all but a few (Gaussian, Cauchy and Levy) has meant that researchers have rarely adopted them in applied work.³ Distributions in the stable class are characterised by the α tail weight parameter that gives a measure of the probability mass located in the tail(s) of the distribution. Under the stable distribution, a finding of $0 < \alpha < 2$ suggests a distribution characterised by theoretically infinite variance and a finding of $0 < \alpha < 1$ suggests also a theoretically infinite mean. A finding of $\alpha = 2$ implies the distribution is Gaussian or normal. The purpose of this research is to provide an initial Australian contribution on box office revenues and to further explore the statistical features of the (potentially) stable box office revenue distribution using a large new Australian data set. This research will also extend the previous work of De Vany and Walls in two ways. Firstly, by reconsidering the suitability of the stable distribution for the ‘cumulative revenue’ distribution when the data set is disaggregated in terms of maximum number of screens on which a film plays in its theatrical life, and secondly, by considering the stable distribution’s appropriateness for the ‘weekly revenue’ and ‘weekly screen average revenue’ distributions.

Previous research of De Vany and Walls (1999, 2004a, 2004b), Lee (1999), Hand (2001) and McKenzie (2005) has solely considered the α tail index parameter in relation to ‘cumulative revenue’ samples of films that were all subject to release on varying numbers of screens (theatres). The potential shortcoming of such an approach is that comparison

³ The application to financial data has received some attention. See, for example, Buckle (1995), Liu and Brorsen (1995), and McCulloch (1997).

of films of differing release types is encountered. For example, in terms of the current data set being considered the sample would include both a wide release film like *Harry Potter – Goblet of Fire* which was released on 552 Australian screens and an art-house film like *Run Lola Run* which was only released on 17 screens. Obviously these films have very different target audiences and earning potential and to compare them in the same ‘cumulative revenue’ sample creates a bias in favour of a lower α estimate – that is to say a distribution with more variance. This paper takes advantage of the fact that the new Australian data set reports weekly screens along with revenue data and the first research question then becomes do the results change when films are grouped as different type of releases? That is, if a film is classified as a small, medium or large release do the conclusions of a non-Gaussian stable distribution still hold? This line of inquiry is motivated by the collection of research written by De Vany and Walls strongly advocating the appropriateness of the stable model for many features of the industry (including box office revenue) and some of the consequential implications they propose of their empirical findings. In particular, De Vany and Walls (2004a) consider an adaptation of a class of model originally proposed by Mandelbrot (1963) that considers the conditional mean in the Paretian tail of the stable distribution by relating it to the conditional revenue of a film and its tail weight parameter. Using the estimated α tail weight parameter, De Vany and Walls then show how (theoretically) a film never loses its revenue earning potential. It is therefore of extreme interest to test the validity of such a strong argument, as the model proposed by De Vany and Walls portrays an almost total inability to predict box office returns.

Beyond assessing the adequateness of the stable model for the disaggregated ‘cumulative revenue’ distribution, the present research also considers the stable model as a potential description of the ‘weekly revenue’ and ‘weekly screen average revenue’ distributions. The exploration of the heavy tailed nature of the data at the ‘weekly revenue’ and ‘weekly screen average revenue’ levels is motivated by the observations of De Vany and Walls (1996, 2004b) who found ‘bifurcation’ points – where films branched into a ‘hit’ or a ‘flop’ – around the fifth to sixth week of films’ lives in their data sets by examining the statistical features of week to week box office revenues signifying the existence of information transmission amongst cinemagoers. They did this in their (1996) paper by showing the interaction of consumers and information feedback that led to the weekly revenue distribution approaching log-log normality at about the sixth week and then in subsequent research (2004b) they were able to show a bifurcation point at the fifth week using regression methodology based on the conditional mean model of Mandelbrot (1963) where movies became ‘hits’ or ‘flops’. There is consequently a motivation to investigate whether any similar sorts of patterns exist in the week to week revenues of the Australian data set and the methodology utilised in this respect is new and novel. The current research also brings new insight into the analysis of box office revenue by directly considering the statistical features of the ‘weekly screen average revenue’ distributions – an area that has so far not been investigated and is only permissible with the current MPDAA Australian data set. This is because the Nielsen EDI and *Variety* figures that are reported for the U.S. and U.K. (by far the most prevalent data source for applied researchers) typically only include *theatre* (location) numbers and not *screen* (engagement) numbers. It is therefore impossible to get an exact screen count when it is

very common for multiplex theatre locations to be screening more than one print per theatre location for a wide release picture.⁴ The benefit of conducting the analysis on ‘weekly screen average revenues’ is therefore that it removes any potential bias that may exist owing to the comparison of small, medium and wide release pictures.

There is also a motivation for this research that goes beyond that of exploring the validity of certain statistical method and robustness. Over the years the multi-billion dollar motion picture industry has at times incurred much scrutiny from the courts and regulatory authorities of the world under accusations such as unfair integrative arrangements and contractual practices – the most famous of these being the Paramount court cases of the late 1940s and early 1950s in the U.S. where the vertically aligned studios were made to divest of their cinema interests.⁵ Australia hasn’t been immune from antitrust issues either, and in the late 1990s the Australian Competition and Consumer Commission (ACCC) undertook a review of the cinema industry after receiving a number of complaints from exhibition interests. These were primarily from small independent exhibitors concerned about the excessive terms being imposed by the large distributors relating to such things as session requirements, minimum run periods and access to first release prints. After these complaints the ACCC recommended the instigation of an industry code of conduct and in 1998 the *Code of Conduct for Film Distribution and Exhibition* was established specifying a number of guidelines relating to the practices of the signatories. The code was initially reviewed in 1999 by the ACCC and is now annually reviewed by an independent auditor and has so far proven to be quite

⁴ For example, the recent Australian block buster *Happy Feet* reported 3,804 theatres for its opening week in the U.S. but its screen/print count was estimated to be in excess of 5,600.

⁵ See De Vany and Eckert (1991).

acceptable to all parties concerned. Although the present study doesn't claim to address these issues directly, a better understanding of the market for motion pictures provides valuable insights that allows one to better appreciate and opionate about legislation of this unique industry.

This paper is organised as follows. Section II provides some background to the performance, structure and practices of the Australian cinema industry – focusing on the distribution/exhibition industries and including a brief discussion of the ‘exhibition contract’ used today in Australia and throughout most of the world. Section III provides an introduction to the data, descriptive statistics and a preliminary discussion relating to the Australian data set. The formal component of this research (Sections IV and V) introduces the stable distribution and the four parameter definition of the distribution is estimated using John Nolan’s program *STABLE v.3.14.02* for the various revenue distributions under consideration. The final section provides a conclusion and discussion of results and reflects briefly on the ACCC’s *Code of Conduct* in light of the observed empirical evidence.

II The Australian Cinema Industry

Industry Performance and Structure

The Motion Picture Distributors Association of Australia (MPDAA) reports that in 2006 Australians went to the movies 4.03 times per head of population. During the 1990s Australia experienced extremely high levels of growth in cinema admissions that were

not matched by any other major market⁶ and even though that growth appears to have slowed somewhat, the cinematic experience is obviously still one of the most popular cultural goods consumed in Australian society with the industry grossing over A\$866m in 2006 – the second highest year on record. The growth in cinema admissions has also been matched by a significant increase in the local infrastructure which has seen the national number of cinema screens almost triple over the last twenty years from 676 in 1986, to 1,964 in 2006 (MPDAA).

There are many companies involved in theatrical distribution and exhibition in the Australian market and the industry has a high degree of integration at both the vertical and horizontal levels. Like most other countries, Australia's theatrical distribution industry is heavily dominated by the major U.S. studios and their associated distribution networks and this is reflected in the large volume of U.S. films released in Australian cinemas relative to films originating from any other country. For example, in 2006 out of a total of 337 Australian domestic releases, 185 originated from the U.S. and only 29 from Australia (MPDAA). The contribution of the Australian theatrical distribution industry is significant and in 2006 the total income reported from film hire⁷ of motion pictures by distributors was A\$333m or 38% of box office receipts (MPDAA). Australia's distribution market structure, like many other countries, is highly concentrated with seven major companies dominating the market. These include the six member companies of the MPDAA (Warner Bros., Paramount Pictures, Universal

⁶ Screen Digest (2000).

⁷ Film hire refers to the portion of box office revenue retained by the distributor after contracted theatrical expenses have been paid.

Pictures,⁸ Buena Vista International, Sony Pictures Releasing⁹ and 20th Century Fox) and Roadshow – a company jointly owned by the exhibition companies Village and Greater Union. The market share of the major distributors has been in excess of 90% in most years with no single company consistently outperforming the others. The remainder of the industry is made up of a number of small Australian distributors which acquire Australian rights to independently produced foreign and local films. These include such companies as Dendy, Icon, Hopscotch, Madman, Palace, Rialto and Hoyts among others. Several of these small distributors have recently formed the Australian Independent Film Distributors Association (AIDA) to lobby on behalf of the group.

The financial contribution of the Australian cinema exhibition industry is also large and in the financial year 1999-2000 (the last year surveyed by the Australian Bureau of Statistics) it was worth over one billion dollars with about 65% of this income generated from gross box office receipts (ABS, 2001). In 2006 there were 494 theatres and 1,964 cinema screens with 55% of these in suburban locations, 40% in country locations and 5% in cities – together total seating capacity was reported to be 467,000 seats (MPDAA). The Australian exhibition industry is also highly concentrated and is dominated by three large companies: Hoyts, Greater Union and Village. These three groups and their subsidiaries control around 50% of Australia's cinema screens and generate around 70% of total box office revenue (ACCC, 1998). Dominance of the exhibition industry, however, may be much greater than these statistics suggest as the competition between

⁸ Universal Pictures and Paramount Pictures formerly operated as United International Pictures (UIP) until December 31, 2006. Paramount Pictures has since commenced distribution of its own films whereas Universal Pictures commence operation in June, 2007.

⁹ Formerly Columbia TriStar.

the major companies has been significantly reduced by an arrangement between two of the three leading exhibitors, specifically Greater Union and Village, to operate multiplex cinemas in suburban and city locations. Similar to the rest of the world, the growth in multiplex cinemas in Australia has been high over the last 15 to 20 years. In 2006, out of 494 theatres, 157 theatres had 5 or more screens and 48 theatres had 10 or more screens (MPDAA).

The Exhibition Contract

The exhibition contract is an interesting and unique contractual arrangement. Film distributors typically charge fees as a percentage of box office revenue rather than a flat fee. The cost of 'film hire' to a cinema is negotiated between the film distributor and the film exhibitor on a per film basis. In Australia film hire terms are typically negotiated between a distributor and exhibitor with consideration to such things as geographical location of the cinema, timing of release, size of release, marketing budgets and (of course) anticipated performance of the film. Distributors often negotiate first with their traditional customers and then make these terms available to similar cinemas. For example, Fox may negotiate firstly with Hoyts locations in the city and then make them available to all similar city theatres but would negotiate different contracts for suburban or regional locations.

There are many variations of the basic exhibition contract but the typical structure used today in Australia establishes a series of claims on the exhibitor's box office revenue that are declining in favour of the exhibitor as the run of the film unfolds. This arrangement

recognises the fact that films typically lose popularity after opening and the opportunity cost of screening alternate titles for the exhibitor. The division of box office revenue is agreed for each week of the run up until a certain week where the share division may become flat. This division of box office usually varies from around 55 to 25 per cent dependant upon the number of weeks a film has been playing and geographic location of the cinema – city cinemas tend to have higher agreed minimum than country cinemas. For example, the first week division may involve a 55/45 split in favour of the distributor, the second week 45/55, the third week 35/65 and weeks beyond the fourth may involve a flat division of 25/75 in the exhibitor's favour.¹⁰ Distributors usually also require that an exhibitor screens a particular title for a certain minimum number of weeks and a certain number of sessions per day depending on the week of the run. A film may receive an extended run if it reaches a certain threshold level known as the 'holdover' clause agreement. In Australia this is commonly assessed as a national level of box office revenue but may also be specific to any particular theatre.

In Australia, the minimum run requirement in the exhibition contract is typically between four and six weeks. Minimum exhibition periods are sometimes described as 'no share' or a film's 'policy'. A typical policy may be one day plus two evening sessions for the first week or two of a run, with more relaxed session requirements towards the end of the required run. This component of the exhibition contract has received some attention from the ACCC in recent years after complaints from small exhibitors objecting to the excessive session requirements for their small sites as compared to those terms offered to larger metropolitan multiplexes. In the *Code of Conduct* outlined by the ACCC in 1998,

¹⁰ Common industry perception is that the typical film retains about 35-40% for the distributor. (MPDAA)

however, no restrictions on session requirements from distributors were instigated. This was interesting given that in 1994 the U.K. Monopolies and Merger Commission Report on the Supply of Films to Exhibition recommended that exhibitors should not be required to commit to a long period of exhibition as a condition of supply of film. The report recommended that a two-week maximum be imposed – exhibitors would be entitled to screen a film for longer at their own discretion, but distributors could not force a longer period.

III The Australian Data – A Preliminary Analysis

The new Australian data set is the largest single sample of films thus far known to be evaluated in the literature and includes 2,429 distinct films released at the Australian box office over the nine year period January 1, 1997 to December 31, 2005 as recorded by the MPDAA. The data reports the ‘cumulative (total) revenue’ that each film earned over its theatrical life and (where available) also includes the week to week breakdown of revenues, screen counts and screen averages. Table 1 (first row) reports descriptive statistics for the total sample of 2,429 films where it can be noted that the calculated mean of ‘cumulative revenue’ is in excess of A\$2.8m with an accompanying median of just under A\$0.6m and a standard deviation of A\$5.4m. The suggestion of a highly right skewed distribution with long ‘leptokurtic’ tails is confirmed with measures of skew and kurtosis being 3.9 and 21.6 respectively for the full sample. The Kernel density plot in Figure 1 also illustrates the extremity of the skew and the existence of the thick right tail with the normal curve overlaid for comparison. The top earning film of the sample,

Titanic, earned in excess of A\$57.6m, more than ten standard deviations away from the calculated mean suggesting the ‘cumulative revenue’ distribution to be far from normal.

[INSERT FIGURE 1 NEAR HERE]

[INSERT TABLE 1 NEAR HERE]

Table 1 also reports descriptive statistics for films that were defined to different size releases (maximum number of screens over life of film) where it is evident that although there is some reduction in the extremity of skew, the right skew does still appear to exist in all defined windows. The second row of Table 1 reports that of the 1,140 films that were subject to a ‘wide release’ (defined as maximum number of screens exceeding 50) there was an estimated mean of A\$5.6m, median of A\$3.2m and standard deviation of A\$6.9m. The suggestion of right skewed distribution with heavy tails was still apparent with the measure of skew and kurtosis 2.9 and 11.4 respectively. For films with maximum week number of screens 50-149, 150-249 and 250+ the right skew is also evident and this is particularly apparent in the window 150-249 which included the extreme outlier of the sample *Titanic* that opened on 227 screens nationwide. To further investigate the nature of the data, similar analysis was considered on the ‘weekly revenue’ distributions for the first ten weeks of films’ lives. The top half of Table 2 reports descriptive statistics for films recorded at week one to week ten and it can be noted that at each week there is an observed right skew similar to the ‘cumulative revenue’ samples where the mean consistently exceeds the median, the standard deviations are large and the values of skew and kurtosis tell a supporting story.

[INSERT TABLE 2 NEAR HERE]

This descriptive evidence, however, may be somewhat premature when it is considered that these statistics are derived from a sample comprising films of differing release types. In this sense, the ‘weekly revenue’ distributions suffer from the same potential bias as the ‘cumulative revenue’ distribution in that the sample is essentially comparing films that played on a varying number of screens in a given week. The bottom half of Table 2 reports descriptive statistics that suggest the range of ‘screen counts’ at each week may also be somewhat skewed (the mean exceeding the median) but not to the same extent as revenue when considering the respective skew and kurtosis measures.

To gain further insight into this potential bias Table 3 reports statistics based on ‘weekly screen average revenues’ for films that went into wide release (maximum screens exceeding 50 at some point in the run), this was done to avoid the art-house, festival and, more importantly, Large Screen Format (LSF) 60mm films that typically play at theatres such as IMAX. These types of films in one sense offer a different cinematic experience that may not be directly comparable with the films of interest in this research (such as 3D films) but, moreover, typically earn excessive screen averages that would bias results in an undesired manner.¹¹ It can be noted in Table 3 that although the right skew is still apparent at all weeks the magnitude of skew in the ‘weekly screen average revenue’ distribution appears to have significantly reduced when the respective measures of skew and kurtosis are compared with those reported in Table 2 for the ‘weekly revenue’ distributions. Further investigation in the next section is intended to explore whether the

¹¹ For example the IMAX film *Everest* had the top opening week screen average of A\$134k compared with the highest ‘wide release’ opening week screen average of A\$46k attained by *Lord of the Rings – Two Towers* which would have ranked 23 in the full sample in terms of opening week screen average.

stable distribution model is still valid for describing the behaviour of the ‘weekly revenue’ and ‘weekly screen average revenue’ distributions.

[INSERT TABLE 3 NEAR HERE]

IV Stable Distributions

The fact that top performing movies tend to take a seemingly unequal share of the available revenue suggests that the box office revenue distribution is one of extreme skew. In fact, the divergence of the mean from the median, reported in the previous section, suggests a strong rightward skew and has led researchers, such as De Vany and Walls (1999, 2004a, 2004b), Lee (1999) and Hand (2001) to explore the characteristics and dynamics of the distribution. De Vany and Walls (1999) show that the rare blockbuster type films dominate the mean and that potential outcomes diverge over all values. They characterise the distribution as a Levy distribution in the family of stable distributions. Similar findings have also been reported by Lee (1999), Hand (2001) and McKenzie (2005). Stable distributions are important as they allow for extreme skewness and heavy tails but the lack of closed formulas for all but a few stable distributions (Gaussian, Cauchy and Levy) has meant that few practitioners have adopted them for use in applied work. In recent times, however, reliable computer programs, such as Nolan’s *STABLE v. 3.14.02*, have been developed that are capable of estimating the parameters of stable densities.

A random variable X has a *stable* distribution if for any positive number A and B , there is another positive number C and real number D such that

$$AX_1 + BX_2 \stackrel{d}{=} CX + D \tag{1}$$

where X_1 and X_2 are independent copies of X , where “ $\stackrel{d}{=}$ ” denotes equality in distribution, i.e. both expressions have the same probability law. This can be seen by using the addition rule for the sum of two independent normally distributed variables, where mean of the sum is the sum of the means and the variance of the sum is the sum of the variances. For example, if the terms on the left hand side of (1) are $N(a\mu, (a\sigma)^2)$ and $N(b\mu, (b\sigma)^2)$ respectively, while the right hand side is $N(c\mu+d, (c\sigma)^2)$, by the addition rule one must have $c^2=a^2+b^2$ and $d=(a+b-c)\mu$. An important characteristic of the stable distribution is its “stability” property. The word stable is used because the shape is stable, or unchanged, under sums of the type described in equation (1). For any stable random variable, X , there is a number $\alpha \in (0,2]$ such that the number C satisfies the following relationship

$$C^\alpha = A^\alpha + B^\alpha. \quad (2)$$

Both (1) and (2) imply that the stable distribution is invariant under aggregation. It is well known that only a few stable distributions have closed formulas for densities and distributions. To overcome this problem it is possible to use a characteristic function uniquely determined by the cumulative distribution function. A random variable X is said to have a stable distribution if there are parameters $0 < \alpha \leq 2$, $\sigma \geq 0$, $-1 \leq \beta \leq 1$, and μ real such that its characteristic function has the following form¹²

$$Ee^{iXt} = \exp[i\mu t + \psi_{\alpha,\beta}(\sigma t)] \quad (3)$$

where

¹² Characteristic functions are discussed in most elementary probability theory textbooks.

$$\psi_{\alpha,\beta}(t) = \begin{cases} -\sigma^\alpha |t|^\alpha \left[1 - i\beta(\text{sign } t) \tan \frac{\pi\alpha}{2} \right], & \text{if } \alpha \neq 1 \\ -\sigma |t| \left[1 + i\beta \frac{2}{\pi} (\text{sign } t) \ln |t| \right], & \text{if } \alpha = 1 \end{cases}$$

and with

$$1 \text{ if } t > 0$$

$$\text{sign } t = 0 \text{ if } t = 0$$

$$-1 \text{ if } t < 0.$$

The four parameters that characterise the distribution are the stability index, α , that governs tail behaviour, the shift parameter, μ , that locates the distribution right or left, the scale parameter, σ , that expands or contracts the distribution around μ , and finally β , that indicates the degree of skewness of the distribution.

V Stable Distribution Results

Equation (3) is estimated using the Maximum Likelihood procedure in Nolan's *STABLE* program. Results of the 'cumulative revenue' analysis are reported in Table 4 and reveal an estimated value of the α tail weight parameter of 1.14 for the full 'cumulative revenue' sample confirming a distribution with finite expected mean but theoretically infinite variance. This conforms to the previous studies cited that have all found varying estimates of $\alpha < 2$ and in the case of Lee (1999) and Hand (2002) have found $\alpha < 1$ suggesting a theoretically undefined expected mean in the upper tail of the distribution. The other estimated parameters confirm the distribution is right skewed ($\beta = 1$) and its location and scale but are of limited interest in the present discussion. The second to fifth rows of Table 4 refine the analysis to estimate (3) for the 'cumulative revenue'

distribution of movies defined subject to their release patterns. The second row considers the sample defined as ‘wide release’ (maximum screens 50 or more) which was chosen to eliminate the cluster of films that fall into the LSF definition discussed in the previous section. Elimination of these types of films reveals an increase in the estimated value of α (1.46) but still well within the range expected a-priori and very close to De Vany and Wall’s common benchmark figure of 1.5 commonly referred to in throughout the literature.¹³ For the disaggregated analysis of ‘cumulative revenue’ of films defined by maximum weekly screens 50-149, 150-249 and 250+, the estimated values of α increased again and were 1.63, 1.56, and 1.72 respectively for the three windows under consideration. It was also noted that the entire distribution essentially shifted rightwards for each sub-sample (as evidenced by larger values of μ - the location parameter) but still retained the property of theoretically infinite variance with the estimate of $\alpha < 2$.

[INSERT TABLE 4 NEAR HERE]

This assessment, in terms of defining films by opening week screens, has so far retained De Vany and Walls’ arguments for the suitability of the stable distribution to describe the disaggregated ‘cumulative revenue’ box office distribution. The fact that the estimates of α did tend to increase when films were redefined in terms of release type, however, does demonstrate that the α parameter is sensitive and not a robust estimate across all films in the sample. This observation becomes particularly relevant if one were to put too much faith in calculations of the expected mean using Mandelbrot’s (1963) Paretian tail distribution adapted by De Vany and Walls (1999, 2004a) where the expected future revenue of a film is described by the equation

¹³ De Vany (2004).

$$E(x | x \geq k) = k \frac{\alpha}{\alpha - 1} \quad (5)$$

where x would represent the expected conditional cumulative revenue given that a film has already achieved k and α represents the tail weight parameter. For example, if a film had already made A\$10m and α was 1.5 the expected revenue of the film would have a conditional mean equal to A\$30m. Obviously such statements are subjective to the value of k and α and the implication is that a film does not exhaust its revenue earning potential. Although perhaps somewhat paradoxical the logic is summarised by De Vany and Walls (2004a, p. 1050): “..movies that make it into the upper tail of the profit (revenue) distribution have been strongly selected from their competitors. The ‘heavy tails’ of the stable distribution imply that probability does not decline rapidly enough for the conditional expectation to converge in the value of the conditioning event...This property seems to capture the intuition behind the term momentum”.

Beyond disaggregating the ‘cumulative revenue’ distribution, the second motivation of this paper is to examine the suitability of the stable distribution as a model of revenue at the ‘weekly revenue’ and ‘weekly screen average revenue’ levels. The results of Table 5 suggest that the ‘weekly revenue’ distributions are also suitably described by the α -stable distribution.¹⁴ The estimates of the α tail parameter all suggest findings in the region of 1 to 1.3 (theoretically infinite variance) with some evidence of the estimate of α falling until week 8 then increasing (see Figure 2) – this observation is paid more attention below.

[INSERT FIGURE 2 NEAR HERE]

¹⁴ The finding of non-normality was also confirmed with Skewness-Kurtosis, Shapiro-Wilk and Shapiro-Francia tests of normality and log-normality.

[INSERT TABLE 5 NEAR HERE]

Recalling that one of the primary objectives of this research was to remove the potential bias of comparing titles showing on differing number of screens (that could potentially bias revenue comparison towards a lower value of α), Table 6 reports the estimates of the four parameter stable distribution for ‘weekly screen average revenue’ distributions of wide release films. That is, the distribution of weekly film revenues divided by their corresponding week number of screens for films that, at some point, were playing on 50 or more screens.

[INSERT TABLE 6 NEAR HERE]

The results of the analysis again suggest that the stable model is suitable for describing all weeks of the ‘weekly screen average revenue’ distributions. It is of more interest to note, however, that all estimates of the α tail parameter are consistently greater than those found relating to the ‘weekly revenue’ distributions reported in Table 5. This result is non-surprising and it is apparent that removing the screen bias does indeed increase estimates of α but not by enough to observe a normal or log-normal distribution.¹⁵ Further observation of Table 6 does reveal an extremely interesting feature in the results as (similar to the results observed above in Table 5 and Figure 2) there does appear to be something of a pattern in the week to week estimates of α . In particular, with some slight exception, the α tail parameter is observed to fall to a minimum value in week 6 and then rise again thereafter suggesting that films develop probability mass most strongly in the early weeks of life before it dissipating in the later weeks. This result can be seen in Figure 3 and can potentially be explained by the role of ‘information transmission’ to

¹⁵ Once again, the rejection of non-normality and non log-normality was also confirmed with Skewness-Kurtosis, Shapiro-Wilk and Shapiro-Francia tests.

create a potential ‘bifurcation’ point when screen averages stop showing such extremities and films become ‘hits’ or ‘flops’.¹⁶ This finding could also be a manifestation of ‘holdover’ clauses that get triggered in the exhibition contract as films performing well at the end of their minimum run have their runs at the box office contractually extended. These two arguments combined do make sense when considered in the following way: A film opens and is contracted for four to six weeks, some open well with high screen averages and continue to maintain strong audiences and high screen averages, whereas others do not open so well and audiences disappear quickly – the fact that screens are still under contract makes for low screen averages with the net result being a distribution with extremes at each end (α falls). Over the early weeks of the run this pattern continues as information is shared and there is increasing probability mass in the tail until the contracted run ends and poor performing films are removed from screens, whereas successful films have their runs extended and are on the ‘hit’ path of the bifurcation mapping (α increases).

[INSERT FIGURE 3 NEAR HERE]

The results of the ‘weekly revenue’ and ‘weekly screen average revenue’ investigations have suggested some important findings. Firstly, the ‘weekly revenue’ and ‘weekly screen average revenue’ distributions do conform to the stable model, and secondly, there is a potential pattern emerging in the weekly evolution of ‘weekly revenues’ and ‘weekly screen average revenues’ that have an economically justifiable explanation. There is, however, a question of interpretation that should be addressed before the discussion is complete and this relates to how one should interpret the estimated α tail parameter in the setting of the ‘weekly revenue’ and ‘weekly screen average’ distributions? The argument

¹⁶ The term ‘bifurcation’ is used following De Vany and Walls (2004b).

of De Vany and Walls (2004a) for the cumulative gross gaining momentum in the conditioning event makes intuitive sense (in a somewhat paradoxical way) when one is considering the Paretian tail (5) of the ‘cumulative revenue’ distribution but not so much when considering the ‘weekly revenue’ or the ‘weekly screen average revenue’ distributions. It doesn’t really makes sense to literally interpret the conditional mean equation (5) when considering these distributions as, by definition, they are truncated events. For example, saying that a film is expected to obtain a week 2 screen average of A\$30,000 given that it has already obtained a week 2 screen average of A\$10,000 is a bit like predicting yesterday’s weather today – the event has been and gone. So perhaps it doesn’t make sense to use the interpretation of the distribution in this sense and perhaps it should not be assessed so literally (as should not the cumulative model – a film will die eventually!). In this context it is strongly suggested that all that should be inferred from these results is insights about the nature of the distribution and the role of momentum in creating the heavy tailed distribution. In observing such a result the logical question then becomes what are the generating forces behind such momentum for which one is left with one simple answer – the extremities of right skew and variance in all of the definitions observed must be the result of a powerful information dynamic.

VI Conclusions and Discussion

This paper has considered the appropriateness of the stable distribution for a large new sample of Australian box office revenue data. The results support the earlier studies of De Vany and Walls to confirm the suitability of the distribution for the Australian sample of the ‘cumulative revenue’ sample. Further, the analysis also revealed that the stable

distribution was appropriate for the disaggregated ‘cumulative revenue’, ‘weekly revenue’ and ‘weekly screen average revenue’ samples. The study has highlighted that although there is an undoubtable self-similarity among the box office revenues definitions considered, estimates of the α tail parameter are far from consistent. It is therefore not appropriate to put too much confidence on inference such as that relating to the conditional Paretian tail model of Mandelbrot (1963) adapted by De Vany and Walls. While such a distribution may be important for better understanding the type of distribution being considered, taking this model too literally is non-advisable. The evidence presented should demonstrate that it is highly important to understand the characteristics of the box office revenue distributions and the role that the extreme events play in its properties. It then becomes important to understand the processes that may lead to such an outcome and the role of information transmission in this result.

This paper has also identified an interesting feature relating to the week to week behaviour of the estimated α tail parameter of the ‘weekly revenue’ and, more profoundly, the ‘weekly screen average revenue’ distributions. It was observed that the α parameter showed evidence of falling in the early weeks and increasing beyond week six. This result can be interpreted as an increase and subsequent decline in the probability mass in the far right tail of the distributions that has a logical explanation considering the interaction and sharing of information between filmgoers and the expiration of minimum run periods and triggering of holdover clauses for strongly performing films. This conclusion is based on the idea that as a film opens a divergence begins as some films achieve high screen averages and others do not. The positive and negative information

being shared increases the divergence as the run unfolds and this is compounded by the minimum run period if it is being enforced by distributors (screen averages fall quickly). By the fourth to sixth week the majority of films have expired minimum run periods and films earn their screens through more market based forces of supply and demand. Successful films also trigger holdover clauses that can gain them extra playing weeks. Overall there is now less divergence and variability in the weekly screen average distributions which is consistent with falling probability mass observed beyond week six. This result and interpretation bears similarities to the findings of De Vany and Walls (1996, 2004b) but with the extra consideration to the role of the exhibition contract in addition to the role of information transmission between consumers.

This paper has demonstrated that motion picture revenues are highly uncertain. It is well known in the industry that most movies don't make significant amounts of money at the box office, but the ones that do seem to take a lion's share. Operating in such an environment then exposes the motion picture distributor and exhibitor to much uncertainty that helps to explain why the industry has sought to organise itself the way it has, and provides strong explanations for the observed behaviour and practises of the industry such as distributors having multiple titles, distributors owning cinemas, cinemas owning distributors, multiplex cinema alliances, etc. In this regard the industry participants rely on portfolio techniques to diversify across contingent outcomes. Within such an environment the ACCC had a difficult task to design a fair and equitable *Code of Conduct for Film Distribution and Exhibition* when it would appear that industry participants are just attempting to protect themselves in a highly uncertain and

unforgiving environment. With the nature of uncertainty that has been documented, it would seem that the typical form of the exhibition contract used today in Australia is a most appropriate mechanism to share risk over the dynamic life of a film. The combination of a declining division of revenue, minimum run period, and hold over clause all act in a way that provides incentives and safeguards that are highly consistent with the uncertain nature of the product. The declining share of revenue (for the distributor) works well to maintain an incentive for the exhibitor to holdover a film given that most films earn the majority of their revenue in the opening and early weeks of their run. The flexibility of weekly divisions is also a useful mechanism for bargaining terms, given that different films have different types of life cycles and also that parties may have different thoughts on how a film is going to perform in relation to itself and against competing films. The minimum run clause is also an appropriate mechanism for ensuring that a film has a good enough chance to capture the word of mouth effect shared by filmgoers that may take some weeks to initialise. In this context it would seem reasonable to stipulate a minimum run of at least four weeks as it has been suggested that a film requires at least this time in order to gain its own life. A shorter upper bound on the minimum run period, such as the two week maximum of the U.K., might not give films time enough to develop an audience and realise its potential. Another feature relevant to the exhibition contract is the provision of the holdover clause that can see a film's run extended based on its revenue performance of early weeks. Such a provision gives a film its maximal chance to realise the extent of its revenue earning potential and, as this is written into a contract before the film opens, may provide the distributor more certainty in its income stream.

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Figure 1
Kernel Density: Cumulative Revenue (\$A)

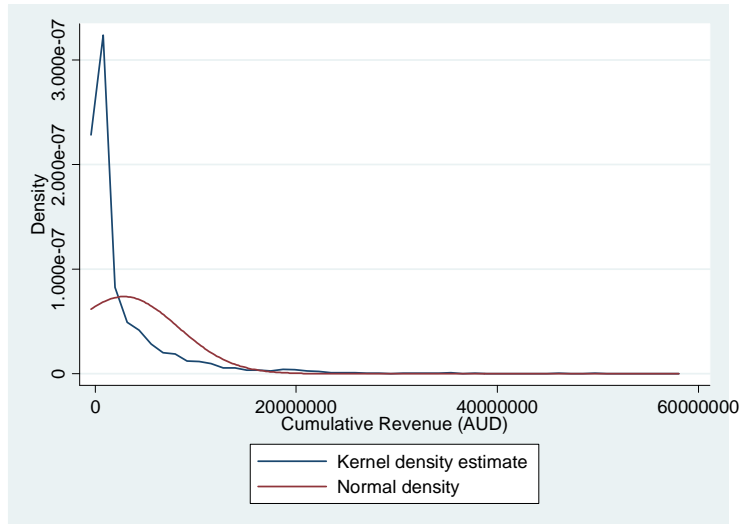


Figure 2
Estimated α for Weekly Revenue Distribution

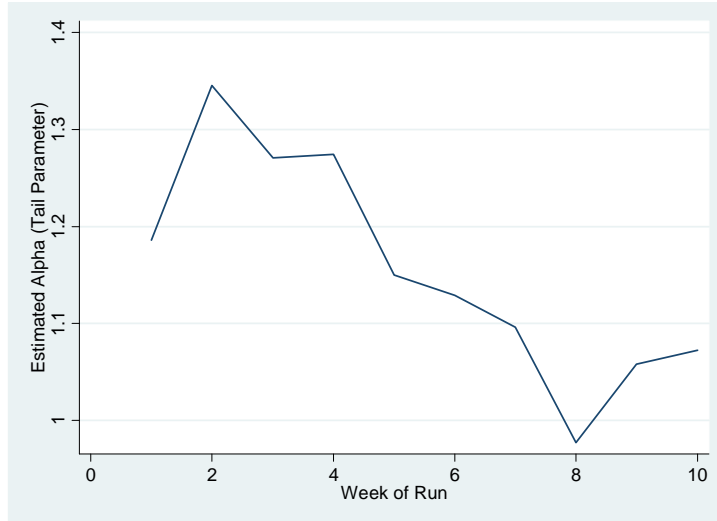


Figure 3
Estimated α for Weekly Screen Average Distribution (Wide Releases)

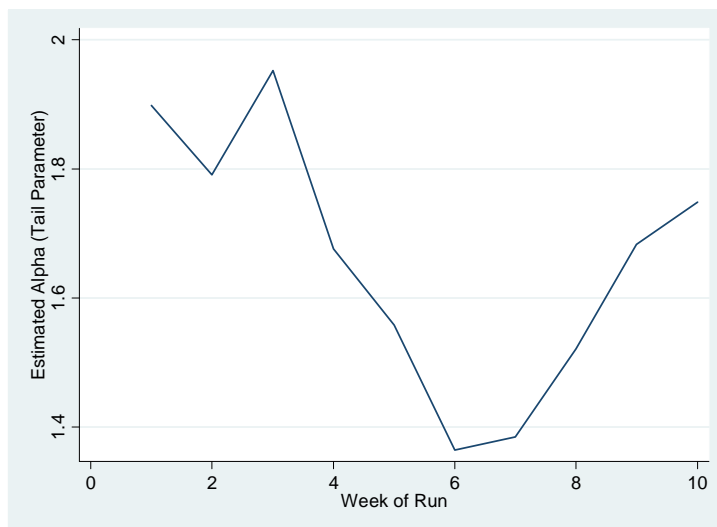


Table 1
Descriptive Statistics: Cumulative Revenue (\$A) by Maximum Screens

Maximum Screens	Obs.	Mean	Median	Std. Dev	Max	Min	Skew	Kurtosis
Full Sample	2,429	2,827,885	593,488	5,405,078	57,645,827	360	3.93	21.57
50+	1,140	5,572,638	3,214,085	6,881,523	57,645,827	25,115	2.86	11.40
50-149	558	1,944,961	1,325,572	2,004,603	14,696,624	25,115	2.37	7.91
150-249	395	5,944,175	4,745,512	4,908,362	57,645,827	502,561	4.11	32.69
250+	187	15,612,674	12,651,013	9,262,994	50,388,327	3,838,901	1.57	2.45

Table 2
Descriptive Statistics: Weekly Revenues (\$A) and Weekly Screens

Week	Obs.	Mean	Median	Std. Dev	Max	Min	Skew	Kurtosis
<i>Revenue</i>								
Week 1	2,302	928,142	195,994	1,765,781	19,617,082	30	4.21	26.08
Week 2	2,025	730,369	209,817	1,214,970	12,464,352	98	3.40	16.84
Week 3	1,806	542,427	170,890	861,940	8,657,317	106	2.98	12.31
Week 4	1,613	385,393	119,789	626,841	6,079,753	37	3.10	12.86
Week 5	1,401	268,140	86,723	472,536	4,489,890	29	3.77	19.43
Week 6	1,224	187,251	63,057	350,325	3,848,849	50	4.49	28.90
Week 7	1,044	134,796	48,085	251,059	2,585,013	72	4.10	22.06
Week 8	874	104,577	38,503	202,802	2,428,437	129	4.70	31.59
Week 9	727	82,753	30,919	168,682	2,431,625	64	6.21	61.42
Week 10	591	72,243	25,505	153,307	2,089,127	39	6.41	61.69
<i>Screens</i>								
Week 1	2,302	86	42	98	552	1	1.30	1.40
Week 2	2,025	97	66	101	552	1	1.11	0.87
Week 3	1,806	98	65	101	552	1	1.04	0.67
Week 4	1,613	90	48	96	552	1	1.08	0.65
Week 5	1,401	78	40	86	435	1	1.20	0.77
Week 6	1,224	65	32	75	419	1	1.39	1.41
Week 7	1,044	53	27	63	380	1	1.73	3.07
Week 8	874	43	22	54	363	1	2.08	5.03
Week 9	727	36	18	45	315	1	2.26	5.96
Week 10	591	31	15	40	244	1	2.36	5.98

Table 3
Descriptive Statistics: Weekly Screen Averages (\$A) (Wide Releases)

Week	Obs.	Mean	Median	Std. Dev	Max	Min	Skew	Kurtosis
Week 1	1,140	9,324	7,928	6,415	46,267	289	1.57	3.88
Week 2	1,120	6,506	5,542	4,858	44,023	107	2.20	9.58
Week 3	1,050	4,660	3,824	3,605	37,393	106	2.19	9.99
Week 4	964	3,466	2,721	2,756	24,779	121	2.27	8.53
Week 5	856	2,675	2,092	2,143	15,175	103	2.28	7.19
Week 6	750	2,197	1,727	1,782	16,807	43	2.62	10.70
Week 7	646	1,900	1,484	1,532	12,764	24	2.51	9.23
Week 8	531	1,765	1,400	1,415	12,266	44	2.73	12.04
Week 9	418	1,700	1,300	1,281	10,132	64	2.10	6.75
Week 10	328	1,663	1,312	1,303	8,562	39	2.04	5.77

Table 4**Stable Parameter Estimates: Cumulative Revenue by Maximum Screens**

Maximum Screens	Obs.	α	β	σ	μ
Full Sample	2,429	1.1363	1	1515460	593488
50+	1,140	1.4558	1	2964980	3214080
50-149	558	1.6333	1	991699	1325570
150-249	395	1.5629	1	2169220	4745510
250+	187	1.719	1	5299550	12651000

Table 5**Stable Parameter Estimates: Weekly Revenues**

Week	Obs.	α	β	σ	μ
Week 1	2,302	1.1857	1	521304	195994
Week 2	2,025	1.3453	1	447701	209817
Week 3	1,806	1.2707	1	315042	170890
Week 4	1,613	1.2744	1	221621	119789
Week 5	1,401	1.15	1	138104	86723
Week 6	1,224	1.129	1	95002	63057
Week 7	1,044	1.096	1	65463	48085
Week 8	874	0.977	1	46410	38503
Week 9	727	1.058	1	37210	30919
Week 10	591	1.072	1	31706	25505

Table 6**Stable Parameter Estimates: Weekly Screen Averages (Wide Releases)**

Week	Obs.	α	β	σ	μ
Week 1	1,140	1.898	1	3757.25	7928
Week 2	1,120	1.791	1	2674.5	5542
Week 3	1,050	1.9523	1	2063	3823.5
Week 4	964	1.6763	1	1429.5	2721
Week 5	856	1.5581	1	1018	2092
Week 6	750	1.3645	1	775	1727
Week 7	646	1.3848	1	674.5	1483.5
Week 8	531	1.5212	1	663	1400
Week 9	418	1.6832	1	680.5	1300
Week 10	328	1.7483	1	680.5	1312.5